# Updating of the Analytical Models of Two Footbridges Based on Modal Testing of Full-Scale Structures

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## Abstract

This paper describes the analytical FE modelling, modal testing and FE model updating of two full-scale pedestrian footbridges. These procedures are well developed in the fields of mechanical and aerospace engineering, and have just started to be applied successfully to large civil engineering structures. The aim of this paper is to describe some of the possible problems that may arise when conducting such experimental exercises, and also to give advice on how these problems may be overcome. In addition, the paper describes the authors' experiences in initially constructing the FE models and conducting the updating itself. Finally, the practical meaning of the changes to the assumed model parameters, resulting from the updating, is discussed. It has been concluded that a period of manual updating should be conducted prior to implementing the automatic updating, as poor starting values may result in the automatic updating software producing unrealistic results.

## 1. Introduction

As civil engineering structures, and in particular pedestrian footbridges, become increasingly slender there is a much greater need for vibrations to be considered at the design stage. Consequently, there is a requirement for the development of guidelines for the accurate modelling of the dynamic behaviour of footbridges.

In the past, the civil engineering sector has made extensive use of simple single degree of freedom (SDOF) models. In many cases, however, SDOF models have proved insufficient for the accurate modelling of slender footbridge structures. The main reason for this is that such models cannot represent the closely spaced modes of vibration which frequently occur in practice. Therefore, it is necessary to use more advanced multi-degree of freedom (MDOF) methods, such as FE analysis, which have the capability to do this. Fortunately, over recent years, FE modelling has become more affordable to civil engineering practitioners. However, with regard to footbridge design, there is a general lack of expertise in FE modelling, particularly with regard to their vibration serviceability performance.

The way forward for developing such expertise is by linking modal testing and FE analysis by the updating of the models of large footbridge structures.

This type of approach has been used for many years in the mechanical and aerospace engineering sectors, but it is only recently that the civil engineering community has begun to adopt this advanced technology.

Considering the benefits of modal testing and FE model updating, a number of modal tests on full-scale structures have been conducted by the University of Sheffield [1].

The aim of the paper is to describe the use of this modal test data in the manual and automatic updating of analytical FE models. This type of analytical exercise is still rare in civil engineering practice. The exercises were done using the ANSYS FE code [2] in conjunction with FEMtools [3], a state-of-the-art updating software which is widely used in the automotive industry. A brief description is given of the role of different criteria, such as the modal assurance criterion (MAC) and the co-ordinate modal assurance criterion (COMAC), which are employed to correlate the test data with the analytical models. In addition, practical suggestions, based on the authors' experiences, are given regarding the steps to be taken when initially constructing a model for updating. Finally, potential problems are discussed which may arise from the incorrect application of this advanced but sensitive technology to typical civil engineering problems.
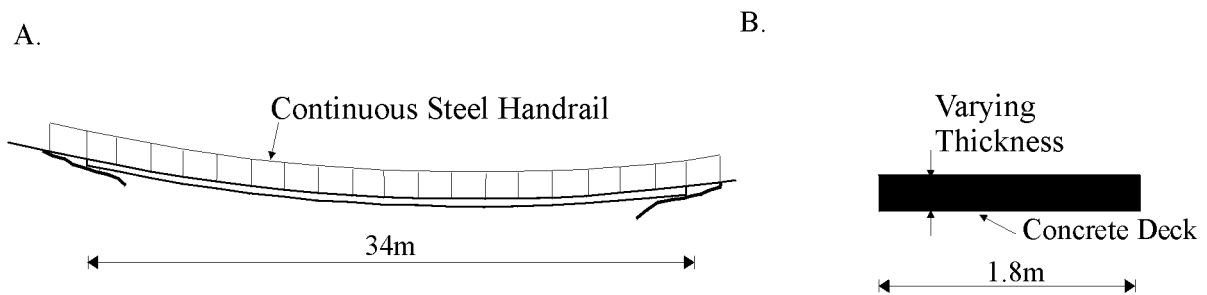
A.

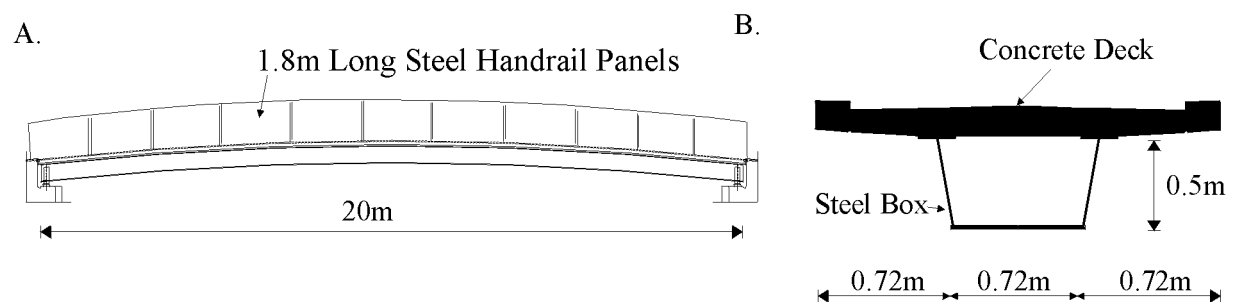Figure 1: Prestressed Ribbon Footbridge (Footbridge 1)

B.

A.

Figure 2: Composite Footbridge (Footbridge 2)

B.

## 2. Test structures

Two full-scale pedestrian footbridges were experimentally tested as part of a doctoral research project at the University of Sheffield between 1994 and 1997 [1]. The data acquired were used to correlate analytical FE models for the purpose of investigating the footbridges' vibration serviceability. The same experimental data is used here to develop new FE models through updating.

The first of these footbridges, a 34m stressed ribbon footbridge, comprises a single span catenary shaped prestressed concrete slab rigidly fixed at the abutments. Steel handrails made of continuous hollow box sections follow the catenary shape of the 1.8m wide deck and are attached firmly to it (Figure 1A). The depth of the slab varies from 380mm at the supports to 160mm at midspan. In addition, there is a 12.5mm topping of asphalt covering the length of the deck. Further details about the cross-section and elevation of this bridge are given in Figure 1.

The second footbridge is a 20m span steel-concrete composite structure. It comprises a precambered 150mm concrete slab on top of a

500mm deep trapezoidal shaped steel box (Figure 2). The structure is supported at each end by an arrangement of elastomeric bearings. It is important to note that the bearings at each end are of a different type. The handrails on the bridge take the form of independent 1.8m long panels made of hollow section components. These are rigidly attached to and follow the shape of the deck. A cross-section and elevation of this structure are given in Figures 2A and 2B.

## 3. FE modelling

When constructing FE models, one of the most important considerations is their intended use. In this case, the models had to be suitable for importation to the FEMtools updating software so that the automatic updating exercises could be carried out. Consequently, only element types which are supported by both the ANSYS FE code and FEMtools could be used in the construction of the model. In addition, when models are going to be correlated with the results from modal test data, it is important to ensure that the nodes of the FE mesh are coincident with the locations of the test points [4].
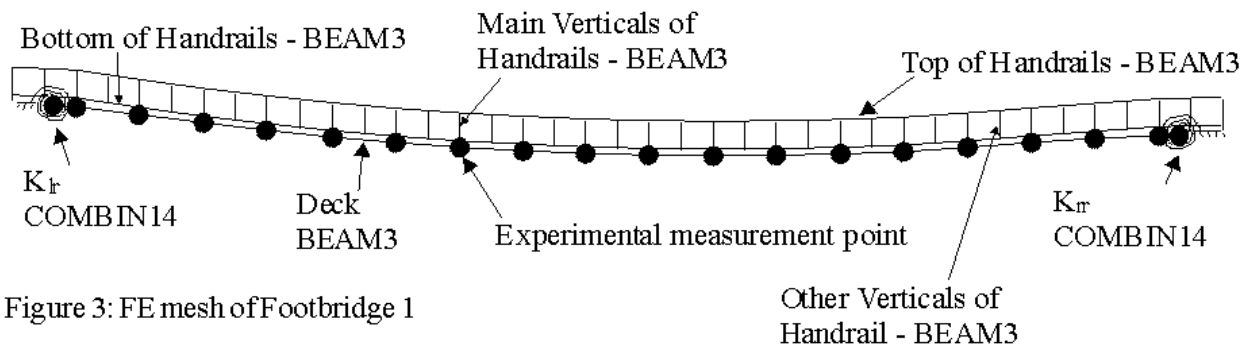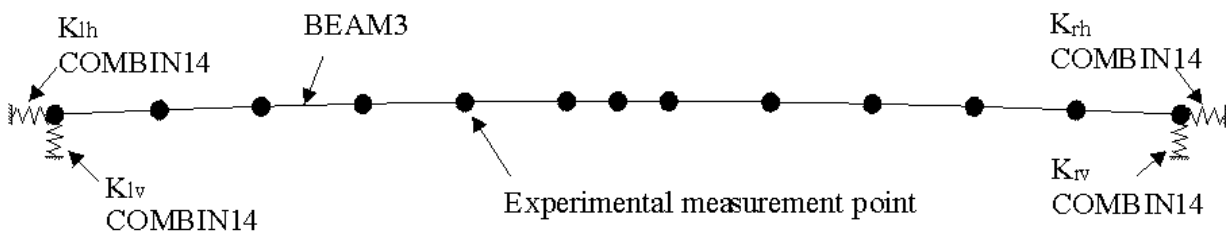
Figure 3: FE mesh of Footbridge 1



Figure 4: FE mesh of Footbridge 2

## 3.1 Modelling of Footbridge 1

A 2-D FE model was constructed using BEAM3 elements available in ANSYS [2]. Ideally, when choosing the type of element, it would have been more appropriate to use BEAM54 elements to model the varying thickness of the footbridge deck [2]. BEAM54 elements allow the input of different section properties, such as area and moment of inertia, at each end of the element. Unfortunately, FEMtools does not support this element type [3], and so a different approach had to be found. The method chosen to model the slab was to 'smear' the geometrical section properties over the length of normal 2D elastic BEAM3 elements. In other words, for each element, the geometrical properties were taken as the mean values of those at the nodes of the element. This is, obviously, not an ideal situation and will introduce some error into the analysis. However, if the elements are small, then the error is reduced. A comparison between the results from using BEAM54 elements and smeared BEAM3 elements showed that the mesh size adopted was sufficiently small so as not to significantly influence the results.

In addition, the mass of the asphalt surface was included as an artificially increased density of the concrete material of the deck.

The supports of the bridge were modelled as two COMBIN14 rotational spring elements (Figure 3) [2]. In addition, vertical and horizontal degrees of freedom were restrained a the supports. Experiences of Pimentel [1] suggest that the continuous handrails on this kind of structure can provide a considerable amount of stiffness to the overall system. Consequently, the handrails were modelled as a frame of elements rather than just as additional mass, the latter being the usual design procedure. The FE mesh and experimental measurement nodes are given in Figure 3.

## 3.2 Modelling of Footbridge 2

Similarly to Footbridge 1, this structure was modelled in ANSYS using 2D elastic BEAM3 elements. The cross-section is constant along the span and hence no averaging of section properties was required. In this case, however, the handrails are a series of independent panels and as such were

| Frequency | Mode shape |
|---|---|
| $f_1 = 2.34$Hz | |
| $f_2 = 3.58$Hz | |
| $f_3 = 4.54$Hz | |
| $f_4 = 6.27$Hz | |
| $f_5 = 8.93$Hz | |
| $f_6 = 11.68$Hz | |

Table 1: Experimental results of footbridge 1

| Frequency | Mode shape |
|---|---|
| $f_1 = 3.64$Hz | |
| $f_2 = 13.21$Hz | |
| $f_3 = 25.47$Hz | |

Table 2: Test results for Footbridge 2

modelled purely as additional mass by artificially increasing density.

The elastomeric bearing supports of the structure were modelled as four COMBIN14 linear elastic spring elements. Two of these represented horizontal stiffnesses and the remaining two vertical stiffnesses. The FE mesh and the experimental test grid are given in Figure 4.

# 4. Modal testing of prototype structures

The testing programmes of the two footbridges included the measurement of their vertical modes of vibration. These tests were conducted using the impulse excitation technique, with an instrumented sledge hammer as the exciter. The collected data were in the form of frequency response functions (FRFs) which were then analysed using MDOF parameter estimation algorithms to obtain natural frequencies, mode shapes and modal damping values. The ICATS suite of programs was used to conduct these analyses [5].

Tables 1 and 2 show the natural frequencies and mode shapes estimated from the collected sets of FRFs. The damping values are not reported here as they were not used in the updating exercises.

# 5. Manual updating

Prior to attempting to update the FE models in FEMtools, it was felt prudent to experiment with the different input parameters by hand. The reasons for this are twofold. Firstly, manual updating enables the analyst to appreciate the possible effect that each

parameter may have on the dynamics of the system. This is extremely important as it provides a basis for judging whether the results obtained from the automatic procedure are sensible. Secondly, unless reasonable starting values of the updating parameters are given, the FEMtools software may have difficulty in improving the correlation between the experiments and the analysis. The results of the manual updating exercises are given in Tables 3 to 6.

## 5.1 Updating parameters - Footbridge 1

| EMA (Hz) | FEA (Hz) | | | | | |
|---|---|---|---|---|---|---|
| | $f_1 = 2.31$ | $f_2 = 3.69$ | $f_3 = 4.7$ | $f_4 = 6.29$ | $f_5 = 8.86$ | $f_6 = 11.8$ |
| $f_1 = 2.34$ | 97.8 | 0.2 | 0.0 | 0.0 | 0.0 | 0.6 |
| $f_2 = 3.58$ | 2.1 | 95.2 | 0.7 | 1.1 | 0.0 | 0.1 |
| $f_3 = 4.54$ | 0.0 | 1.8 | 94 | 3.2 | 0.1 | 0.1 |
| $f_4 = 6.27$ | 1.2 | 0.0 | 0.1 | 97.5 | 0.0 | 0.4 |
| $f_5 = 8.93$ | 0.0 | 0.1 | 0.7 | 0.7 | 96.5 | 0.1 |
| $f_6 = 11.7$ | 0.1 | 0.0 | 0.0 | 1.1 | 0.5 | 91.9 |

Table 3: MAC values (%) after manual updating of Footbridge 1

The most difficult aspect of conducting updating exercises is the choice of which parameters to change. In this case, it was decided to use the Young's modulus of elasticity of the deck and of the handrails, and the rotational stiffnesses of the

supports. The initially assumed values for the manual updating parameters are presented in Table 4. In addition, Table 4 contains the values of the same parameters, which were found to give the best agreement after the manual updating. The initial rotational stiffness for the supports was chosen arbitrarily whilst the E values for concrete and steel were those typically used in static calculations.

| Parameter | Initial Value | Best Value |
|---|---|---|
| $E_{steel}$ | 200 kN/mm$^2$ | 200 kN/mm$^2$ |
| $E_{concrete}$ | 30 kN/mm$^2$ | 42.0 kN/mm$^2$ |
| $K_{lr}$ | $1 \times 10^8$ Nm/rad | $2 \times 10^8$ Nm/rad |
| $K_{rr}$ | $1 \times 10^8$ Nm/rad | $2 \times 10^8$ Nm/rad |

Table 4: Manual parameter changes of Footbridge 1.

## 5.2  Updating parameters - Footbridge 2

| EMA (Hz) | FEA (Hz) | | |
|---|---|---|---|
| | $f_1 =$ 3.67 | $f_2 =$ 13.25 | $f_3 =$ 26.82 |
| $f_1 =$ 3.64 | 94.7 | 0.3 | 4.6 |
| $f_2 =$ 13.21 | 0.2 | 99.1 | 0.1 |
| $f_3 =$ 25.47 | 1.5 | 0.0 | 99.2 |

Table 5: MAC values (%) after manual updating of Footbridge 2

| Parameter | Initial Value | Best Value |
|---|---|---|
| $E_{steel}$ | 200 kN/mm$^2$ | 200 kN/mm$^2$ |
| $E_{concrete}$ | 30 kN/mm$^2$ | 32 kN/mm$^2$ |
| $K_{lh}$ | $1.5 \times 10^6$ N/m | $4.3 \times 10^8$ N/m |
| $K_{lv}$ | $8 \times 10^6$ N/m | $9.3 \times 10^7$ N/m |
| $K_{rh}$ | $1.4 \times 10^6$ N/m | $3.8 \times 10^8$ N/m |
| $K_{rv}$ | $1.5 \times 10^6$ N/m | $1.5 \times 10^8$ N/m |

Table 6: Manual Parameter changes of Footbridge 2

The parameters chosen for manual updating in this model were the vertical ($K_{lv}$ and $K_{rv}$) and horizontal ($K_{lh}$ and $K_{rh}$) stiffnesses of the left and right supports and the E values of the concrete and steel. The initial values for the support stiffnesses were based upon the values reported in the manufacturers sales literature [1]. The values were different for the two

supports as the elastomeric material differed. These initial values and those which gave the best correlation from the manual updating are presented in Table 6.

# 6. Automatic updating

| EMA (Hz) | FEA (Hz) | | | | | |
|---|---|---|---|---|---|---|
| | $f_1 =$ 2.34 | $f_2 =$ 3.57 | $f_3 =$ 4.63 | $f_4 =$ 6.28 | $f_5 =$ 8.79 | $f_6 =$ 11.7 |
| $f_1 =$ 2.34 | 97.7 | 0.3 | 0.0 | 0.2 | 0.0 | 0.4 |
| $f_2 =$ 3.58 | 2.4 | 95.1 | 0.1 | 1.3 | 0.1 | 0.0 |
| $f_3 =$ 4.54 | 0.0 | 0.1 | 97.0 | 2.3 | 0.0 | 0.1 |
| $f_4 =$ 6.27 | 0.9 | 0.1 | 0.3 | 97.9 | 0.0 | 0.0 |
| $f_5 =$ 8.93 | 0.0 | 0.0 | 0.2 | 0.6 | 98.3 | 0.1 |
| $f_6 =$ 11.7 | 0.2 | 0.0 | 0.0 | 0.4 | 0.4 | 95.4 |

Table 7: MAC values (%) after automatic updating of Footbridge 1

| EMA (Hz) | FEA(Hz) | | |
|---|---|---|---|
| | $f_1 =$ 3.67 | $f_2 =$ 13.06 | $f_3 =$ 25.47 |
| $f_1 =$ 3.64 | 94.6 | 0.2 | 4.6 |
| $f_2 =$ 13.21 | 0.2 | 98.6 | 0.0 |
| $f_3 =$ 25.47 | 1.3 | 0.0 | 96.7 |

Table 8: MAC values (%) after automatic updating of Footbridge 2

The parameters that were chosen for automatic updating in FEMtools were the same as for the manual exercises. However, for Footbridge 2, the E values for steel and concrete were updated as a single E value corresponding to a composite cross section. In addition, it was found that if natural frequencies were the only updating criterion considered, the mode shapes that resulted were not always sensible. Consequently, the MAC values of the analytical and measured mode shape pairs were also used as an updating criterion. This provided some consideration of the mode shapes in the analysis. However, the recommended confidence level in mode shapes, is a

factor of ten lower that that assigned to the natural frequencies [3]. Also, it is wise to physically examine the mode shapes and assign lower confidence factors if the shapes appear to be erratic. This happens frequently when using hammer testing on large civil engineering structures due to environmental noise.

The results from the automatic updating analyses for the two structures are presented in Tables 7 to 10, and Figures 5 and 6.

# 7. Discussion

For most civil engineering structures, unlike those for mechanical and aerospace engineering, the designs are one-off. This requires designers to ensure that the structure performs satisfactorily, first time. Hence, the immediate benefit of updating models of existing civil structures, which are already designed and built, is not as obvious as, for example, in automobile design. It is important to stress here that the main advantage of this exercise to civil engineering is not in the improvements of existing prototypes and their numerical models. The principal benefit is more reliable modelling to assist in future designs of similar structures. This is particularly important in the design of footbridges where it is common for similar types of construction to be used repeatedly. Consequently, increased knowledge, from the updating of FE models, based upon prototype testing could be extremely valuable to designers in the future.

## 7.1 General observations

It can be seen (Tables 2, 5, 7 and 8), that the difference between the analytical natural frequencies and the experimental (EMA) results was reduced by the automatic updating. However, the MAC values were not significantly improved. The probable reason for this is that the EMA mode shapes are not estimated sufficiently well.

As an example, in Table 2, the first mode shape ordinates for Footbridge 2 do not lie on a smooth curve, as would be expected. This is almost certainly due to errors in the experimental data. Consequently, the MAC values will be limited by the accuracy of the predicted EMA mode shapes. Also, although the frequencies may have changed, the intrinsic shape of the mode may not be that different. Therefore, only small changes in MAC could be expected. In addition, Figures 5 and 6 show the COMAC for Footbridge 2. These are calculated using a combination of modes 2 & 3 and 1, 2 & 3

respectively. It is clear that the inclusion of the erratic EMA-1 generally worsens the COMAC values.

This highlights one of the potential problems of the blind application of automatic updating procedures. If poor experimental data is used then the updated parameters may be meaningless, and such a validated model may not be suitable for further usage. This is a good example showing why a close examination of the experimental data is strongly recommended prior to using it in the updating [6]. As previously suggested, this is particularly important when using noisy modal testing data collected from testing large civil engineering structures in open space environments. Again, this is a good argument for conducting manual updating exercises first, as a requirement, for any unrealistic change of the parameters is likely to be better understood.

## 7.2 Interpretation of parameter changes in Footbridge 1

It is important to realise that the validity of the updated parameter values, in a practical sense, depends on how many of the possible updating parameters have been used. For example, in these exercises, changes in Young's modulus represent global changes of stiffness. This, however, could be attributed to actual differences in the assumed area or mass density.

Table 9 lists the required parameter changes in Footbridge 1, and shows that the predicted rotational stiffnesses are much greater than originally assumed. Having in mind that the supports of a catenary shaped prestressed ribbon footbridge are typically assumed to be either pinned or fixed, this finding may help in the future FE modelling of similar structures.

Further examination of Table 9 shows that the major differences are in the parameters representing the components of the handrails. This makes sense from a modelling point of view, as the contribution of these non-structural elements to the dynamic system is extremely difficult to predict. In a practical sense, the values of E for the verticals of the handrails are unrealistic. However, it is important to realise exactly what these updated parameters represent. Each vertical element represents a number of vertical bars and, as such, the value of E given in Table 9 represents the contribution that all of these bars make to the global stiffness of the dynamic system. The increase in stiffness of the vertical elements is a
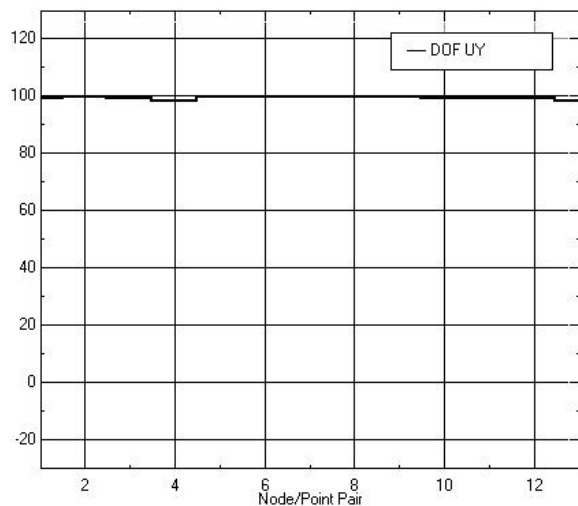
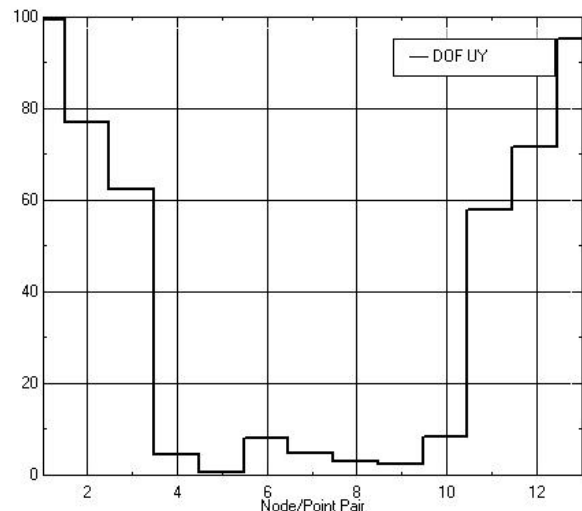Figure 5: COMAC of Footbridge 2 – EMA-2 & EMA-3



Figure 6: COMAC of footbridge 2 – EMA-1, 2 & 3

consequence of the fact that they are effectively acting as ties between the two levels of horizontal elements (decking and handrails). This means that the system is acting somewhere between the two following idealised conditions. The first condition is where the horizontal elements act entirely independently and the second is where there is an absolute rigid connection between the two levels of horizontal elements.

The E value of the concrete is predicted as being 37.4 kN/mm$^2$, which is consistent with similar values recommended by Wyatt, for use in the vibration design of building floors [7]. This relatively high value is also confirmed by the authors' own experience in conducting FE model correlation exercises on long span prestressed concrete floors.

## 7.3 Interpretation of parameter changes in Footbridge 2

Table 10 shows the parameter changes resulting from the automatic updating of Footbridge 2. It is apparent that the largest changes from the automatic updating are in the stiffnesses of the supports. Again this is logical as it is the support stiffnesses that are the most difficult to predict. From a practical point of view, it appears that the dynamic stiffness of the elastomeric materials are significantly higher in the horizontal direction than the values suggested in the manufacturers sales literature, although the differences in the vertical direction are much less. Unfortunately, no information is available as to how the values in the manufacturers literature were determined.

| Parameter | Initial Value | Best Value |
|---|---|---|
| E $_{Top\ of\ Hand\ Rails}$ | 200 kN/mm$^2$ | 219 kN/mm$^2$ |
| E $_{Bottom\ of\ Hand\ Rails}$ | 200 kN/mm$^2$ | 124 kN/mm$^2$ |
| E $_{Main\ Verticals\ of\ Hand\ Rails}$ | 200 kN/mm$^2$ | 378 kN/mm$^2$ |
| E $_{Other\ Verticals\ of\ Hand\ Rails}$ | 200 kN/mm$^2$ | 216 kN/mm$^2$ |
| E$_{concrete}$ | 42.0 kN/mm$^2$ | 37.4 kN/mm$^2$ |
| K$_{lr}$ | $2.00 \times 10^8$ Nm/rad | $5.84 \times 10^8$ Nm/rad |
| K$_{rr}$ | $2.00 \times 10^8$ Nm/rad | $7.65 \times 10^8$ Nm/rad |

Table 9: Automatic updating parameter changes – Footbridge 1

As a result, the reasons for the difference could not be established. Nevertheless, it is probable that differences such as specimen dimensions or confinement conditions could explain the discrepancies. In addition, the relatively small movements at the supports during modal testing could have an effect since such materials may be expected to have a higher stiffness at lower strain levels.

The E value of the composite section after automatic updating has been determined to be 206 kN/mm$^2$. Assuming that the moment of inertia of the

composite section and $E_{steel}$ are unchanged, then the corresponding $E_{concrete}$ value is 35.8 kN/mm$^2$.

| Parameter | Initial Value | Best Value |
|---|---|---|
| $E_{composite}$ | 200 kN/mm$^2$ | 206 kN/mm$^2$ |
| $K_{lh}$ | $4.3 \times 10^8$ N/m | $4.24 \times 10^8$ N/m |
| $K_{lv}$ | $9.3 \times 10^7$ N/m | $7.63 \times 10^7$ N/m |
| $K_{rh}$ | $3.8 \times 10^8$ N/m | $4.24 \times 10^8$ N/m |
| $K_{rv}$ | $1.5 \times 10^8$ N/m | $8.86 \times 10^7$ N/m |

Table 10: Automatic updating parameter changes – Footbridge 2

## 8. Conclusions

The procedure of FE model updating using modal test data of full-scale structures has be applied successfully to two large pedestrian footbridges. The analyses have highlighted the following general points:

1.  The blind application of the automatic updating procedures built in purposely developed software can easily produce meaningless results in the case of civil engineering applications.
2.  Potential problems arising from the use of poor experimental data and a failure to appreciate whether the parameter changes are realistic, can be avoided if the analyst performs an initial period of manual updating and only uses good quality experimental data.
3.  Unless the initial input parameters are sufficiently close to the final values, the highly sensitive updating software may not find a good solution. In addition, if only natural frequencies are used for updating, then unrealistic mode shapes may be predicted.

## Acknowledgements

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